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SHRINKAGE TESTIMATORS AND THEIR APPLICATIONS FOR THE RECIPROCAL OF SHAPE PARAMETER OF PARETO DISTRIBUTION

B. N. Pandey
Department of Statistics, Banaras Hindu University, Varanasi - 221 005, India
Email: profbnpandey@yahoo.co.in

A. K. Srivastava and H. O. Tiwari
Department of Statistics, Banaras Hindu University, Varanasi - 221 005, India
Email: skanandcap@yahoo.co.in

SUMMARY

In this paper, the estimation for the reciprocal of the shape parameter of the Pareto distribution $(1/a)$ has been considered. If $(1/a_0)$ be the prior guess value of $(1/a)$ then shrunken estimator for $(1/a)$ has been proposed and its properties have been studied. For socio-economic data n is usually large, the normal approximation to test statistics can be applied (see Rahman et al. (1997)) which gives equal tail areas on either side of uniformly most powerful unbiased test. By taking normal approximations to test statistics a shrunken testimator and preliminary testimator for $(1/a)$ have been proposed and its properties are discussed.

Keywords and phrases: Preliminary Testimator, Shrinkage Testimator, Linex Loss Function, Modified Linex Loss Function, Numerical Integration and Gauss-Laguerre Quadrature Formula.

1 Introduction

The guess value can be incorporated in the estimation procedure to increase the precision of the estimate through preliminary testing or shrinkage techniques (see Bancroft (1944) and Thompson (1968)). Saleh (2006) considered the theory of preliminary test and Stein_type estimations. Hirano (1977), Pandey and Singh (1980), Pandey (1979), Singh et al. (1973), Mehta and Srinivasan (1971), Pandey and Srivastava (2001), Pandey et al. (2004), Pandey et al. (2005) have investigated preliminary testimators and shrunken testimators for different distributions. Sclove et al. (1972) showed the non-optimality of a Preliminary Testimator (P.T.) for the mean of a normal distribution. Davis and Arnold (1970) proposed shrunken estimators for variance using P.T. for the mean of a normal distribution. Hirano (1984) specified the best value of level of significance as 16%. Kamboo et al. (1990) proposed shrunken testimator for the mean of exponential distribution under type II censored data. Hogg (1974) used a different weight k which is more conservative than the above in the sense that if test statistics is near to boundary of the critical region one takes $k \simeq 1$ while Hogg's take $k = 1/2$. This suggests that the use of test statistics for the P.T. in the construction of weight function k is more reasonable than fixed or predetermined value of k. Conerly and Hardin (1991) suggested that shrunken estimators perform quite poorly in probability nearness (P.N.) criterion. Thomas (1976) derived the reciprocal moments of linear combinations of exponential variates. The resultant formula is used to obtain the moments of quantile and other similar estimators for the shape parameter of a Pareto distribution. The general formula for the reciprocal moments is shown to be potentially useful in linear models and in studying models of the variation in the rate of births in a pure birth process.

Vilfredo Pareto (1897) introduced Pareto distribution, which is commonly used to study the higher income distribution. Arnold (1983) and Longe (1978) discussed the history and sociological implications of Pareto distribution. Davis and Foldstein (1979) viewed Pareto distribution as a potential model for life testing experiments. Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from a Pareto distribution with probability density function (p.d.f.)

$$f(x, a, \sigma) = a\,\sigma^a x^{-(a+1)}, \;\; x \geq \sigma, \;\; a > 0, \tag{1.1}$$

where a and σ are shape and scale parameters, respectively. The maximum likelihood estimates (m.l.e.) of a and σ are

$$\hat{\sigma} = \mathrm{x}_{(1)} = \min(x_1, x_2, \ldots, x_n) \quad \text{and} \quad \hat{a} = \left[\frac{1}{n}\sum_{i=1}^{n} \log \frac{x_i}{x_{(1)}}\right]^{-1}, \tag{1.2}$$

which are jointly sufficient and consistent estimates of a and σ are respectively. Malik (1970) derived the distribution of m.l. estimators and showed that they are independently distributed. Sakesena and Johnson (1984) obtained the unique minimum variance unbiased estimators for shape parameters based on complete sufficient statistics (see Baxter (1980), Likes (1969)). If we make the transformation $y = \log x$, the resulting distribution has the p.d.f. given by

$$f(y, \log\sigma, a) = ae^{-a(y - \log\sigma)}, \;\; y \geq \log\sigma, \; a > 0, \tag{1.3}$$

which is displaced exponential distribution with shifted location parameter $\log\sigma$ and scale parameter $(1/a)$, respectively. The maximum likelihood estimator of $\log\sigma$ and $(1/a)$ are given by

$$\log\hat{\sigma} = y_{(1)} = \log x_{(1)} \quad \text{and} \quad \left(\frac{1}{\hat{a}}\right) = \left[\frac{1}{n}\sum_{1}^{n}\left(y_i - y_{(1)}\right)\right] = P_1. \tag{1.4}$$

We concentrate on the conditional distribution of order statistics $(y_{(2)}, y_{(3)}, \ldots, y_{(n)})$ of the sample given $y_{(1)}$ when the first failure occur. Using the classical transformation

$$Z_i = (n - i + 1)\left(y_{(i)} - y_{(i-1)}\right), \;\; i = 2, \ldots, n \tag{1.5}$$

more specifically, $Z_1 = ny_{(1)}$, $Z_2 = (n-1)(y_{(2)} - y_{(1)})$, $Z3 = (n-2)(y_{(3)} - y_{(2)})$ and so on. It is clear that Z_1 and $T = \sum_{i=2}^{n} Z_i$ are independent and $(Z_1 - n\log\sigma)/a_0$ has $\Gamma\left(1, (aa_0)^{-1}\right)$.

We have $Z_2, Z_3, \ldots, Z_n$ as i.i.d. exponential with mean $(1/a)$. Thus, problem reduces to estimating $(1/a)$ on the basis of random samples of size $(n-1)$ from a population with p.d.f. $p\,(z \mid a) = a\,e^{-az},\ a > 0$. If $1/a_0$ be the prior guess value of $1/a$, the test statistic for $H_0 : a_0/a = 1$ is based on $T/a_0 = \sum_{i=2}^{n} Z_i/a_0$. We know that $(2aT/a_0^2) \sim \chi^2_{2(n-1)}$. Therefore we have

$$E\left[\frac{T}{(n-1)a_0^2}\right] = \frac{1}{a} \tag{1.6}$$

Sakesena and Johnson (1984) obtained minimum mean squared error (MMSE) estimator for a in the class

$$P_2 = c_1 \left\{\frac{(n-2)a_0^2}{T}\right\} \quad \text{is} \quad P_2^* = \left\{\frac{(n-3)a_0^2}{T}\right\},$$

which has

$$MSE\,(P_2^*) = \frac{a^2}{n-2}. \tag{1.7}$$

The MMSE estimator for $(1/a)$ in the class

$$P_3 = c_2 \frac{T}{(n-1)a_0^2} \quad \text{as} \quad P_3^* = \frac{T}{na_0^2}$$

which has

$$MSE\,(P_3^*) = \frac{1}{na^2}. \tag{1.8}$$

We have considered the shrunken estimator P_{2s} for $(1/a)$ and expression for mean square error and Risk under modified Linex loss function has been obtained in Section 2. Comparison with respect to P_3^* is made. The shrunken estimator is preferable if $H_0 : a_0/a = 1$ is accepted otherwise the MMSE estimator may be preferred. The shrunken testimator for $(1/a)$ is

$$P_{2e} = \begin{cases} \frac{1}{a_0}\left[1 - \frac{\left(1-\frac{T}{(n-1)a_0}\right)^3}{\left(1-\frac{T}{(n-1)a_0}\right)^2 + \frac{T^2}{(n-1)^3 a_0^2}}\right], & \text{under } H_0 : \frac{a_0}{a} = 1 \\ \frac{T}{na_0^2}, & \text{otherwise.} \end{cases} \tag{1.9}$$

Muniruzzaman (1957) obtained a test statistics for $\mathrm{H}_0\colon (a_0/a) = 1$ against $\mathrm{H}_1\colon (a_0/a) \neq 1$ as $T^* = (2aT/a_0^2) \sim \chi^2_{2(n-1)}$. The rejection rule can be obtained by using chi-square statistics. Reject H_0 if $\chi^2_{2(n-1)} \leq l_1$ or $\chi^2_{2(n-1)} \geq l_2$, where l_1 and l_2 are the unbiased partitioned or equal tail partitioned values of chi-square at $\alpha\%$ level of significance with $2(n-1)$ degree of freedom.

In socio-economic data, the values of n are usually large and one can use the normal approximation to (T/a_0) which justify the use of a test with equal tail areas in either side. The statistics

$$y = \frac{(aT/a_0^2) - (n-1)}{\sqrt{n-1}} \sim N\,(0,1) \tag{1.10}$$

and is symmetrical. The hypothesis $H_0 : a = a_0$ cannot be rejected if

$$P\left[-1 \leq \frac{(T/a_0) - (n-1)}{Z_{\frac{\alpha}{2}}\sqrt{n-1}} \leq 1\right] = 1 - \alpha \tag{1.11}$$

The shrunken factor may be taken as

$$0 \leq \left[\frac{(T/a_0) - (n-1)}{Z_{\frac{\alpha}{2}}\sqrt{n-1}}\right]^2 \leq 1 \tag{1.12}$$

which provides the shrunken estimator for $(1/a)$ as

$$P_3 = \frac{1}{a_0}\left[1 + \frac{\{(T/a_0) - (n-1)\}^3}{(n-1)^2 Z^2_{\frac{\alpha}{2}}}\right] \tag{1.13}$$

We also have the proposed shrunken testimator for $1/a$ as

$$P_{3e} = \begin{cases} \frac{1}{a_0}\left[1 + \frac{\{(T/a_0)-(n-1)\}^3}{(n-1)^2 Z^2_{\frac{\alpha}{2}}}\right], & \text{if } 1 - \frac{Z_{\frac{\alpha}{2}}}{\sqrt{n-1}} \leq \frac{T}{(n-1)a_0} \leq 1 + \frac{Z_{\frac{\alpha}{2}}}{\sqrt{n-1}} \\ \frac{T}{na_0^2}, & \text{otherwise.} \end{cases} \tag{1.14}$$

The expressions for MSE of the above testimator P_{3e} has been obtained in Section 3 and comparison with respect to the estimator $T/(na_0^2)$ has been made. If $1/a_0$ be the estimate of $1/a$ and MLE for $1/a$ is P_1, the Shrunken estimator for $1/a$ may be given as

$$P_1^* = \frac{1}{a_0} + k_1\left(P_1 - \frac{1}{a}\right), \quad 0 \leq k_1 \leq 1.$$

The estimated value of k_1 for which MSE (P_1^*) will be minimum, is

$$\hat{k}_{1m} = \frac{\left(1 - \frac{T}{(n-1)a_0}\right)^2}{\left(1 - \frac{T}{(n-1)a_0}\right)^2 + \frac{T^2}{(n-1)^3 a_0^4}}. \tag{1.15}$$

More precisely, the preliminary shrunken testimator for $\frac{1}{a}$ is defined as

$$P_{4e} = \begin{cases} \frac{1}{a_0} + \hat{k}_{1m}\left(P_1 - \frac{1}{a_0}\right), & \text{if } \quad 1 - \frac{Z_{\alpha/2}}{\sqrt{n-1}} \leq \frac{T}{(n-1)a_0} \leq 1 + \frac{Z_{\alpha/2}}{\sqrt{n-1}} \\ \frac{T}{na_0^2} & \text{Otherwise.} \end{cases} \tag{1.16}$$

Varian (1975) proposed a very useful asymmetric loss function known as Linex loss function which may be given as

$$L(\Delta) = b\left[e^{p\Delta} - p\Delta - 1\right], \qquad b > 0, \;\; p \neq 0, \Delta = \hat{\mu} - \mu.$$

Where p is shape parameter and b is scale parameter. LINEX was originally proposed because of its flexibility in capturing the asymmetry in a loss structure where such a property is warranted, $L(\Delta)$ rises exponentially when $\Delta < 0$ (under-estimation) and almost linearly when $\Delta > 0$(over-estimation), for small values of $|p|$, $L(\Delta) = \frac{bp^2}{\theta^2}\left(\hat{\theta}-\theta\right)^2$ is a symmetric function (Basu and Ebrahimi (1991)).

Despite the flexibility and popularity of the LINEX loss function for the location parameter estimation it appears to be not suitable for the estimation of the scale parameter and other quantities (Basu and Ebrahimi (1991) and Parsian and Sanjani Farsipour (1993). For these reasons Basu and Ebrahimi (1991) defined a modified LINEX loss;

$$L(\Delta^*) = b\left[e^{p\Delta^*} - p\Delta^* - 1\right],$$

where the estimation error Δ is expressed by $\left((\hat{\theta}/\theta)-1\right)$. Such modification does not change the characteristics of Varian's LINEX loss. This loss function has been considered by Zellner (1986). Pandey et al. (1996) considered the Bayes estimation of shape parameter of Pareto distribution under Linex-Loss function (c.f. Pandey and Rai (1992), Pandey and Srivastava (2001), Pandey et al. (2004), Pandey et al. (2005)). In Section 4 we have proposed the shrunken testimator for $(1/a)$ under Linex loss function and properties of this testimator have been discussed.

2 Shrinkage Estimator for $(1/a)$ and Its Properties

The proposed shrunken estimator for $(1/a)$ may be defined as

$$P_{2s} = \frac{1}{a_0}\left[1 - \frac{\left(1-\frac{T}{(n-1)a_0}\right)^3}{\left(1-\frac{T}{(n-1)a_0}\right)^2 + \frac{T^2}{(n-1)^3a_0^2}}\right]. \tag{2.1}$$

We have

$$\begin{aligned} MSE(P_{2s}) &= E\left(P_{2s}-\frac{1}{a}\right)^2 \\ &= \frac{1}{\Gamma(n-1)a_0^2}\int_0^\infty\left[1-\frac{\left(1-\frac{\delta u}{(n-1)}\right)^3}{\left(1-\frac{\delta u}{(n-1)}\right)^2+\frac{\delta^2u^2}{(n-1)^3}}-\delta\right]^2 e^{-u}u^{n-2}du, \quad (2.2) \end{aligned}$$

where $u = aT/a_0^2$, $\delta = a_0/a$.

The numerical integration can be obtained by 10 point Gauss-Laguerre quadrature formula. We know that Z_1 and $T = \sum_{i=2}^n Z_i$ are independent random variables such that $(Z_i - n\log\sigma)/a_0$ has $\Gamma\left(1,(a_0a)^{-1}\right)$ distribution, which indicates that Z_1 and $T = \sum_{i=2}^n Z_i$

are independent random variables such that $(Z_i - n\log\sigma)/a_0$ has $\Gamma\left(1,(a_0 a)^{-1}\right)$ distribution. The relative efficiency of the estimator P_{2e} with respect to MMSE estimator is defined as

$$REF\,(P_{2s}, P_3^*) = \frac{MSE\,(P_3^*)}{MSE\,(P_{2s})} \tag{2.3}$$

The relative efficiency for the above estimators for different values of $n = 15(5)25$, $\delta = 0.5(0.25)1.5$ have been computed and are presented in Table 1 and we observe that

1. The relative efficiency is maximum near $\delta = 1$
2. The relative efficiency decreases as the sample size increases
3. The effective interval of δ decreases as n increases
4. The estimator is preferable if $0.75 \leq \delta \leq 1.25$ and sample size is small

The risk of P_{2s} under modified linex loss after little bit algebra is

$$\begin{aligned} R\,(P_{2s}) &= b\Bigg[\frac{\exp\{-p(1-(1/\delta))\}}{\Gamma(n-1)} \\ &\quad \times\left\{\int_0^\infty e^{-u}u^{n-2}\exp\big\{-p\,\phi(u,\delta,n)/\delta + \phi(u,\delta,n)\big\}\right\}du + (p-(p/\delta)-1)\Bigg] \end{aligned} \tag{2.4}$$

where $u = aT/a_0^2$, $\delta = a_0/a$ and

$$\phi(u,\delta,n) = \frac{\left(1-\frac{u\delta}{n-1}\right)^3}{\left(1-\frac{u\delta}{n-1}\right)^2 + \frac{u^2\delta^2}{(n-1)^2}}.$$

The numerical integration can be obtained by 10 point Gauss-Laguerre quadrature formula.

3 Shrunken Testimator for $(1/a)$ and Its Properties using Normal Approximations

The shrunken testimator for $(1/a)$ (after) taking normal approximations to test statistics is

$$P_{3e} = \begin{cases} P_3, & \text{if } 1 - \frac{Z_{\frac{\alpha}{2}}}{\sqrt{n-1}} \leq \frac{T}{(n-1)a_0} \leq 1 + \frac{Z_{\frac{\alpha}{2}}}{\sqrt{n-1}} \\ \frac{T}{na_0^2}, & \text{otherwise,} \end{cases} \tag{3.1}$$

where

$$P_3 = \frac{1}{a_0}\left[1 + \frac{(n-1)\left\{\frac{T}{(n-1)a_0} - 1\right\}^3}{Z_{\frac{\alpha}{2}}^2}\right] \tag{3.2}$$

is the shrunken estimator. We observe that P_3 has better performance if $\frac{a}{a_0} \approx 1$ and level of significance is small.

$$\begin{aligned} MSE\left(P_{3e}\right) &= E\left(P_{3e} - \frac{1}{a}\right)^2 \\ &= E\left[P_3^2 \,\middle|\, (n-1) - Z^*_{\frac{\alpha}{2}} \le \frac{T}{a_0} \le (n-1) + Z^*_{\frac{\alpha}{2}}\right] \ \Pr\left\{(n-1) - Z^*_{\frac{\alpha}{2}} \le \frac{T}{a_0} \le (n-1) + Z^*_{\frac{\alpha}{2}}\right\} \\ &\quad + E\left[\left(\frac{T}{na_0^2} - \frac{1}{a}\right)^2 \,\middle|\, \frac{T}{a_0} \le (n-1) - Z^*_{\frac{\alpha}{2}} \bigcap \frac{T}{a_0} \ge (n-1) + Z^*_{\frac{\alpha}{2}}\right] \\ &\quad \times \Pr\left\{\frac{T}{a_0} \le (n-1) - Z^*_{\frac{\alpha}{2}} \bigcap \frac{T}{a_0} \ge (n-1) + Z^*_{\frac{\alpha}{2}}\right\}, \end{aligned} \tag{3.3}$$

where $Z^*_{\frac{\alpha}{2}} = Z_{\frac{\alpha}{2}}\sqrt{(n-1)}$. The expression can be written as

$$\begin{aligned} MSE\left(P_{3e}\right) &= \frac{1}{a^2\sqrt{2\pi}} \int_{b_1}^{b_2} \left[\left\{\frac{1}{\delta\,(n-1)^2\, Z^2_{\frac{\alpha}{2}}} \left(\left\{\left(y\sqrt{n-1} + (n-1)\right)\delta - (n-1)\right\}^3 + (1/\delta - 1)\right)\right\}^2 \right. \\ &\qquad \left. - \left(\frac{y\sqrt{n-1} - 1}{n}\right)^2\right] \exp\left(-y^2/2\right)\, dy + \frac{1}{na^2} \end{aligned} \tag{3.4}$$

where

$$y\sqrt{(n-1)} = \left\{\tfrac{aT}{a_0^2} - (n-1)\right\} \sim N(0, n-1), \quad b_1 = \tfrac{1}{\delta}\left\{\sqrt{n-1} - Z_{\frac{\alpha}{2}}\right\} - \sqrt{n-1}$$

$$b_2 = \tfrac{1}{\delta}\left\{\sqrt{n-1} + Z_{\frac{\alpha}{2}}\right\} - \sqrt{n-1}, \qquad \delta = \tfrac{a_0}{a}.$$

The numerical integration can be obtained by 16–point Gauss–quadrature formula.

$$RE\left(P_{3e}, P_3^*\right) = \frac{MSE\left(P_3^*\right)}{MSE\left(P_{3e}\right)} \tag{3.5}$$

The relative efficiency of the above estimator for different values $n = 30, 40, 50$, $\delta = 0.75(0.25)1.25$, $\alpha = 0.01, 0.05, 0.16$, have been computed and are shown in Table 2. We observe that

1. The relative efficiency is maximum near $\delta = 1$
2. The effective interval for δ is $0.75 \le \delta \le 1.25$
3. If $\delta \le 0.6$, the maximum gain is obtained for moderate values of α (16%)

4 Preliminary Testimator for $(1/a)$ under Linex Loss Function

We know that MMSE estimator for $(1/a)$ in the class P_3 is $T/(na_0^2)$ with $\mathrm{MSE}\,(P_3^*) = 1/(na^2)$. The modified LINEX loss function may be defined as

$$L\,(\Delta*) = b\left[e^{p\Delta*} - p\Delta * -1\right]\;, \qquad \Delta* = \left(\frac{\mathrm{cT}}{1\!\!/\mathrm{a}}{-1}\right), \quad \mathrm{A} \neq 0, \quad \mathrm{b} > 0$$

$$L\,(\Delta*) = b\left[e^{p(\mathrm{caT}-1)} - p\,(\mathrm{caT}-1) - 1\right] \tag{4.1}$$

$$R\,(\Delta*) = b\left[e^{-p}\left(1 - a_0^2pc\right)^{-(n-1)} - (n-1)a_0^2pc + p - 1\right]. \tag{4.2}$$

The value of c for which risk will be minimum is

$$\mathrm{c}_{\min} = \frac{1}{a_0^2p}\left(1 - e^{-\frac{p}{n}}\right). \tag{4.3}$$

The improved estimator for $(1/a)$ under Linex function is

$$\mathrm{P}_2' = \frac{1}{a_0^2A}\left(1 - e^{-\frac{A}{n}}\right)T \tag{4.4}$$

$$MinR\,(P_2') = b\left[A - n\left(1 - e^{-\frac{A}{n}}\right)\right] \tag{4.5}$$

If A $\rightarrow$ 0

$$\mathrm{P}_3' = \frac{T}{na_0^2}$$

which is MMSE estimator for $(1/a)$. Thus, MMSE estimator $\frac{T}{na_0^2}$ is inadmissible under linex loss function.

If a_0 is prior value of a then the preliminary testimator (Bancroft, 1944) for $(1/a)$ is

$$P_{4e} = \begin{cases} \frac{1}{a_0} & \text{if } \mathrm{H}_0 : a = a_0 \text{ is accepted} \\ \frac{1}{a_0^2A}\left(1 - e^{-\frac{A}{n}}\right)T & \text{otherwise} \end{cases} \tag{4.6}$$

The test statistics for $\mathrm{H}_0 : a = a_0$ is

$$y = \frac{\frac{aT}{a_0^2} - (n-1)}{\sqrt{n-1}} \sim N\,(0,1)$$

Here

$$R\left(P_{4e}\right)=bE\left[\left\{e^{A\left(\frac{1}{\delta}-1\right)}-A\left(\frac{1}{\delta}-1\right)-1\right\}\Big/(n-1)-\sqrt{(n-1)}Z_{\frac{\alpha}{2}}\le\frac{T}{a_0}\le(n-1)-\sqrt{(n-1)}Z_{\frac{\alpha}{2}}\right]$$

$$\Pr\left[(n-1)-\sqrt{(n-1)}Z_{\frac{\alpha}{2}}\le\frac{T}{a_0}\le(n-1)-\sqrt{(n-1)}Z_{\frac{\alpha}{2}}\right]$$

$$+b\left[\left\{e^{A\left\{\frac{1}{\delta^2A}\left(1-e^{-\frac{A}{n}}\right)-1\right\}}-A\left\{\frac{1}{\delta^2A}\left(1-e^{-\frac{A}{n}}\right)-1\right\}-1\right\}\Big/\left\{\begin{array}{l}\left(\frac{T}{a_0}\le(n-1)-\sqrt{(n-1)}Z_{\frac{\alpha}{2}}\right)\bigcap\\ \left(\frac{T}{a_0}\ge(n-1)+\sqrt{(n-1)}Z_{\frac{\alpha}{2}}\right)\end{array}\right\}\right]$$

$$\Pr\left[\left(\frac{T}{a_0}\le(n-1)-\sqrt{(n-1)}Z_{\frac{\alpha}{2}}\right)\bigcap\left(\frac{T}{a_0}\ge(n-1)+\sqrt{(n-1)}Z_{\frac{\alpha}{2}}\right)\right] \tag{4.7}$$

$$R\left(P_{4e}\right)=b\delta\sqrt{\frac{n-1}{2\pi}}\int_{b_1}^{b_2}\left[e^{-A}\left\{e^{A/\delta}-e^{\left(1-e^{-\frac{A}{n}}\right)+\left(\sqrt{n-1}y+(n-1)\right)}\right\}-\frac{A}{\delta}+\left(1-e^{-\frac{A}{n}}\right)\right]e^{-\frac{y^2}{2}}dy$$

$$+b\delta\sqrt{n-1}\left[e^{-A+(n-1)\left(1-e^{-\frac{A}{n}}\right)+\frac{1}{2}\left(1-e^{-\frac{A}{n}}\right)^2(n-1)}-\left(1-e^{-\frac{A}{n}}\right)(n-1)+A-1\right] \tag{4.8}$$

where

$$y=\frac{\frac{aT}{a_0^2}-(n-1)}{\sqrt{n-1}}\sim N(0,1),$$

$$b_1=\frac{1}{\delta}\left\{\frac{1}{\sqrt{n-1}}-Z_{\frac{\alpha}{2}}\right\}-\sqrt{n-1},b_2=\frac{1}{\delta}\left\{\frac{1}{\sqrt{n-1}}+Z_{\frac{\alpha}{2}}\right\}-\sqrt{n-1}\text{ and }\delta=\frac{\mathrm{a}_0}{\mathrm{a}}$$

If A→ 0, the Linex loss reduces to squared error loss and the proposed testimator is

$$P'_{4e}=\begin{cases}\frac{1}{a_0} & \text{if } 1-\frac{\mathrm{Z}_{\frac{\alpha}{2}}}{\sqrt{\mathrm{n}-1}}\le\frac{T}{(n-1)a_0}\le1+\frac{\mathrm{Z}_{\frac{\alpha}{2}}}{\sqrt{\mathrm{n}-1}}\\ \frac{T}{na_0^2} & \text{otherwise}\end{cases} \tag{4.9}$$

The properties of the testimator P'_{4e} can be studied.

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Table 1: $R.E.(P_{2s}, P_3*)$

δ / n	0.5	0.75	1.0	1.25	1.5
15	1.0004	1.3265	1.5708	1.3687	1.0007
20	0.9864	1.2371	1.5944	1.2697	0.8967
25	0.6348	1.1467	1.6328	1.0347	0.4962

Table 2: $R.E.(P_{3e}, P_3*)$

α	δ / n	0.5	0.75	1.0	1.25
0.01	30	0.9431	1.7568	2.6439	1.8687
	40	1.0134	1.5832	2.2431	1.5934
	50	0.8625	1.2346	2.0749	1.3241
0.05	30	0.9523	1.4262	1.8477	1.5092
	40	1.0346	1.2315	1.6804	1.3607
	50	0.9325	1.1468	1.5436	1.1659
0.16	30	0.9814	1.3413	1.7258	1.4345
	40	1.0071	1.1517	1.5531	1.2015
	50	0.9636	1.0945	1.4153	1.0945

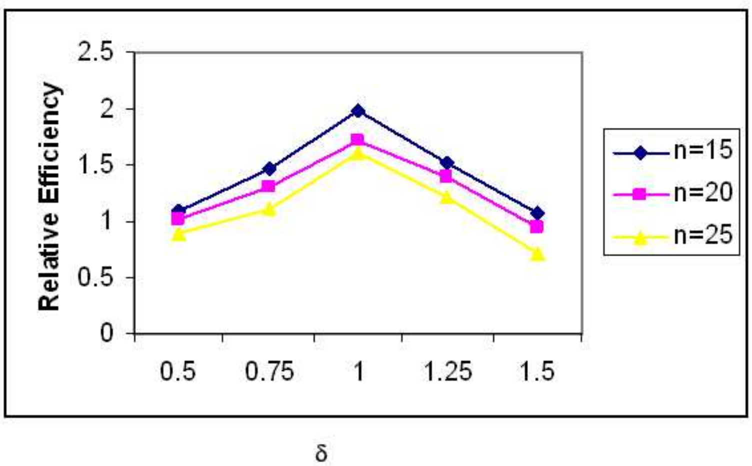

Figure 1: $R.E.(P_{2s}, P_3*)$

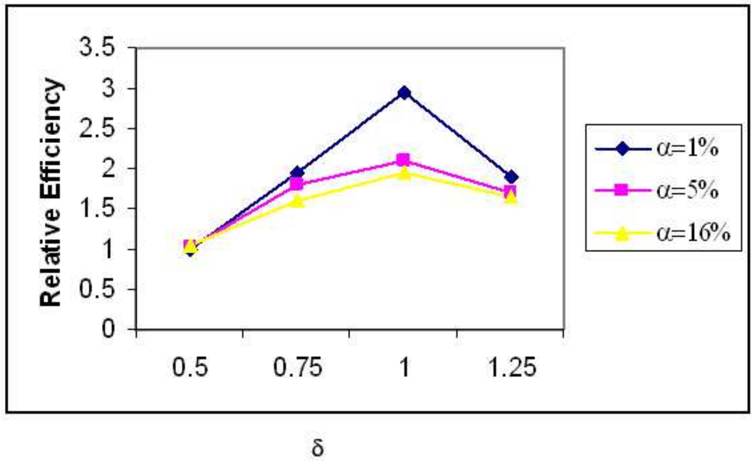

Figure 2: $R.E.\,(P_{3e}, P_{3}*)$ for n=30

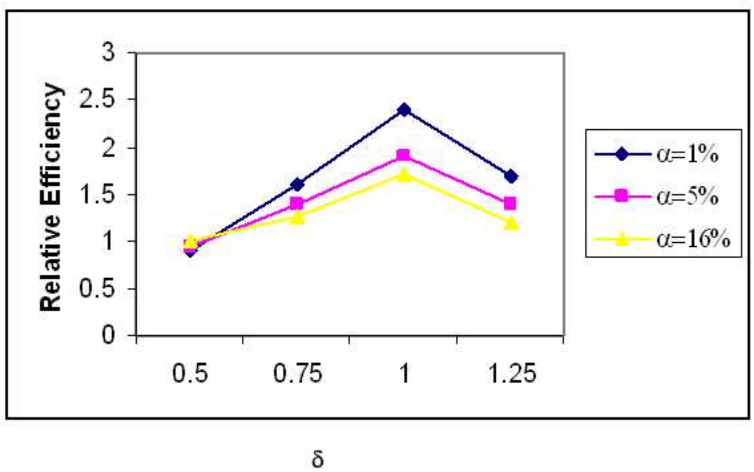

Figure 3: $R.E.\,(P_{3e}, P_{3}*)$ for n=40

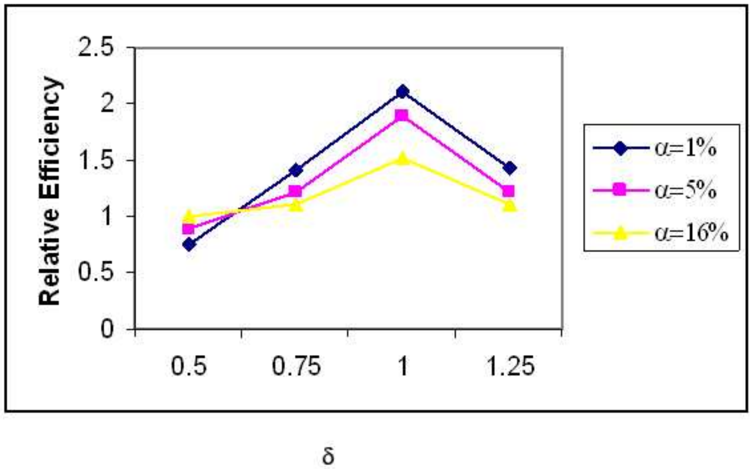

Figure 4: $R.E.(P_{3e}, P_3*)$ for n=50

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