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ON TESTING ORDER RESTRICTED MEAN RESIDUAL LIFE FUNCTIONS UNDER CENSORING

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SUMMARY

In this paper we consider the problems of testing H_0 : a mean residual life (MRL) is exponential versus H_1 : it is new better than used in expectation (NBUE), but not exponential first and then H_0 versus H_2 : it is decreasing MRL (DMRL), but not exponential. The paper gives a nice chronological summary of the tests developed for these two classes both for the complete data and incomplete data cases. We have developed a new test for each class under censoring. Both tests are shown to be consistent in their classes. We have derived the asymptotic distributions of our test statistics.

Keywords and phrases: Hypothesis tests; Order restricted estimator; Kaplan-Meier estimator; Mean residual life; Censoring.

AMS Classification: ...

1 Introduction

Let X be a non-negative random variable denoting the time of occurrence of an event of the subjects with the distribution function (DF) $F,\ F(t) = P(X \leq t)$. Let $S(t) = 1-F(t)$ be the corresponding survival function (SF). The mean residual life is an important biometric function, which is a conditional concept. The mean residual life function (MRLF) at time $t,\ M(t)$, is the average remaining life among those population members who have survived until time t. Throughout we assume that the mean of X, $\mu = \int_0^\infty S(u)du < \infty$. Then $M(t)$ is defined by

$$M(t) = E[X - t|X > t] = \frac{1}{S(t)} \int_t^\infty S(u)\, du\, I[S(t) > 0], \tag{1.1}$$

where the indicator function of the set $A,\ I(A)(t) = 1$ if $t \in A$ and 0 if $\notin A$.

The importance of the MRLF is due to its wide range of applications. Actuaries apply MRL to setting rates and benefits for life insurance companies. In the social sciences we can use the MRLF for modeling the life-length of wars and strikes. The MRLF occurs naturally in areas such as biomedical sciences, optimal disposal of an asset, renewal theory, and reliability. More information on applications of the MRL function can be found in Gross and Clark (1975) and Kuo (1984).

It has been found in various studies that if M is known to be monotone then improved estimate of M may be obtained for that particular class so that it has the monotonic property of the class. This is why there are many tests developed for a monotone M in the literature. A brief description of the tests in the chronological order of their occurrence is given below.

Let $X_1^{(0)}, \ldots, X_n^{(0)}$ be independent random variables with a common continuous DF, F and let $U_1, \ldots, U_n$ be independent positive random variables with possibly discontinuous and defective common DF, G that are independent of the $X_i^{(0)}$'s. The estimator of the MRLF M considered here is based on the censored date (X_i, δ_i) for $1 \leq i \leq n$, defined by

$$X_i = \min(X_i^{(0)}, U_i), \quad \text{and} \quad \delta_i = I(X_i^{(0)} \leq U_i).$$

Let $X_{(i)}s$ be the order statistics of $X_i s$. Then, for $t \in [X_{(k)},\ X_{(k+1)}),\ k = 0, 1, \ldots, n-1$, with the usual convention that $X_{(n)}$ is uncensored whether it is censored or not, the Kaplan-Meier estimate, by Kaplan and Meier (1958), of the survival function is given by

$$S_n(t) = \prod_{X_{(i)} \leq t} \left[1 - \frac{1}{n-i+1}\right]^{\delta_{(i)}},\ \mathrm{t}\ \geq 0. \tag{1.2}$$

Yang (1977) derived an estimator of M simply by plugging in the Kaplan-Meier estimate of $S,\ S_n$, in the definition of M in the equation (1.1):

$$M_n(t) = \frac{1}{S_n(t)} \int_t^\infty S_n(s)\, ds.$$

Yang has also shown that M_n is a strongly uniformly consistent estimator of M on $[0, b]$ for all $b < \tau_F$, where $\tau_F = \inf\{t : F(t) = 1\} \leq \infty$. Kumazawa (1987) has also considered the estimation of the MRL under censoring, and proved that M_n is a strongly uniformly consistent estimator of M on $[0,\ T]$, where $T = \max(X_1, \ldots, X_n)$, and also proved the following theorem for the order unrestricted MRL process.

Theorem 1. *Suppose the distributions F and G satisfy the conditions $(i)\sqrt{n}\ h(T) \to 0$ in probability as $n \to \infty$, $(ii) \int_0^{\tau_H} h^2(t)\ dC(t) < \infty$, where $H = 1 - S(1-G)$ denotes the distribution of X_1, $\tau_H = \sup\{t : H(t) < 1\}$ and so on, $h(t) = \int_t^{\tau_F} S(s)\ ds$, and $C(t) = \int_0^t [S^2(s)\{1 - G(s^-)\}]^{-1}\, dF(s)$, then the unrestricted MRL process $\{B_n(t) \equiv \sqrt{n}\,[M_n(t) - M(t)] : 0 \leq t \leq T\}$ converges weakly in $D[0, \tau_H]$ as $n \to \infty$ to a Gaussian process $\{B(t) : 0 \leq t \leq T\}$ with zero mean and covariance function $cov\{B(s),\ B(t)\} = \{S(s)S(t)\}^{-1} \int_t^{\tau_H} h^2(u)\, dC(u) \quad (s \leq t)$. The Gaussian process $\{B(t) : 0 \leq t \leq T\}$ is given by $B(t) = Z(t)\ M(t) + \int_t^{\tau_H} Z(s)\, dh(s)/S(t)$, where $\{Z(t) : 0 \leq t \leq T\}$ is a Gaussian process with zero mean and covariance function $cov\{Z(s),\ Z(t)\} = C\{min(s,\ t)\}$.*

For the incomplete data, Koul and Susarla (1980) have derived a test for H_0 versus H_1, where $H_0 : S(x) = exp\left(-\frac{x}{\lambda}\right),\ x \geq 0,\ \lambda > 0$ (unknown), and $H_1 : F$ is NBUE, not an exponential. In the case of complete data, tests for the preceding problem have been considered by Hollander and Proschan (1975) and Koul (1978). Koul and Susarla (1980) have considered the parameter

$$J(F) \equiv \mu^{-1} \int_0^\infty \int_0^y S(x)dx\, dF(y) = \frac{\int_0^\infty S^2(x)dx}{\int_0^\infty S(x)dx}.$$

Clearly $F \in H_0$ implies $J(F) = \frac{1}{2}$. On the other hand $J(F) = \frac{1}{2}$ and F continuous and NBUE implies that F is in H_0. The more $J(F)$ differs from $\frac{1}{2}$ the more there is evidence in favor of an $F \in H_1$. This test statistic is a nontrivial analog of the test statistic developed by Hollander and Proschan (1975), which is suitable for the uncensored data. With their estimator $\hat{S_n}$ (modified form of the Kaplan-Meier estimator, S_n, of S), their test statistic is

$$T_n \equiv \frac{\int_0^{L_n} \hat{S}_n^2(x)dx}{\int_0^{L_n} \hat{S}_n(x)dx},$$

where L_n is a deterministic constant depending on the null d.f. F, and censoring d.f. G with $L_n \uparrow \infty$ as $n \to \infty$. They have also given the null asymptotic distribution of T_n under certain condition. This test is to reject H_0 if T_n is large. The alternative form of the test statistic T_n and its computational formulas can be seen in Koul and Susarla (1980). Kumazawa (1986) has presented a class of test statistic for the same test which includes the test statistics proposed in Koul and Susarla (1980) as a special case. Kumazawa (1986a) has proposed a class of test statistics

$$J_n(\kappa) \equiv \frac{\int_0^T K(t)S_n(t)dt}{\mu_n \int_0^T \kappa(t)S_n(s)ds},$$

where the weight function κ is nonnegative, and right continuous, $\mu_n = \int_0^T \hat{S}_n(t)dt$, and $K(t) = \int_0^t \kappa(s)ds$. Kumazawa (1986a) and (1986b) have obtained asymptotic normality of J_n under some regularity conditions and discussed the asymptotic efficiencies under proportional censoring model.

Hollander and Proschan (1975) have derived tests of the null hypothesis that the underlying MRL is exponential, versus the alternative that it has a DMRL, not exponential for the complete data. Yuan, Hollander, and Langberg (1983) have generalized the same test for the randomly censored data and presented the efficiency loss due to the presence of censoring. They have considered the parameter

$$\triangle(F) = \int \int_{s<t} S(s)S(t)\{M(s) - M(t)\}dF(s)\, dF(t),$$

which is the same as the parameter used by Hollander and Proschan (1975) in the uncensored case. To form the test statistic, they have replaced F with $F_n = 1 - S_n$, where S_n is the Kaplan-Meier estimate of S as in (2). Since $\triangle(F_n)$ is not scale-invariant, in order to make it scale-invariant they have used the test statistic

$$V_n^c = \frac{\triangle(F_n)}{\mu_n},$$

where $\mu_n = \int_0^\infty S_n(u)\,du$, which is a consistent estimator of μ, under the assumption that the mean is finite and under suitable regularity on the amount of censoring. The computational form and other details of the test statistic V_n^c can be seen on Chen, Hollander, and Langberg (1983). Some other tests for DMRL using censored data can be found in Lim and Koh (1996) and Lim and Park (1993).

In Section 2 we derive the test statistic Θ_n for the NBUE test, prove the consistency of the test and derive the limiting distribution of Θ_n. In Section 3 we derive the test statistic $\sqrt{n}\,\Theta_n^*$ for the DMRL test, prove the consistency of the test, and derive the limiting distribution of $\sqrt{n}\,\Theta_n^*$. Conclusions are given in the section 4.

2 Testing the exponential distribution versus the NBUE distribution

In this section we consider the problem of testing $H_0 : F(t) = 1 - e^{-\lambda t}$ for $t \geq 0,\ \lambda > 0$ unspecified versus $H_1 : F$ is in NBUE, not exponential. This is equivalent to $H_0 : M(t) = \frac{1}{\lambda}$ for all $t \geq 0,\ \lambda > 0$ unspecified, but fixed versus $H_1 : M(0) \geq M(t)$ and $M(t) \neq \frac{1}{\lambda}$ for all $t \geq 0$.

2.1 The test statistic and its properties

We consider the following parameter as a measure of the deviation from H_0 to H_1 :

$$\Theta = \sup_{0 \leq s \leq t} \frac{M(0) - M(s)}{\sigma(t)}.$$

From theorem 1 we have

$$E[B(t)]^2 \equiv \sigma^2(t) = \{S(t)\}^{-2} \int_t^\infty h^2(u)\,dC(u).$$

Gill (1980) has shown that the empirical estimate, $\sigma_n^2(t)$ of $\sigma^2(t)$ is uniformly consistent on $[0,\ b]$ for any b with $S(b) > 0$. Kumazawa (1987) also noted that when S is exponential, $\sigma(t)$ is nondecreasing. The assumption that $\sigma(t)$ is nondecreasing is satisfied because for $S = e^{-t}$,

$$\frac{d}{dt}\sigma^2(t) \quad = \quad e^{2t}\left[2\int_t^\infty \frac{e^{-u}du}{\bar{G}(u)} - \frac{e^{-t}}{\bar{G}(t)}\right] \geq e^{2t}\left[2\int_t^\infty \frac{e^{-u}du}{\bar{G}(t)} - \frac{e^{-t}}{\bar{G}(t)}\right] = 2e^{2t}\,\frac{e^{-t}}{\bar{G}(t)},$$

when S is exponential, irrespective of G. Using this and the uniform consistency of $\sigma_n^2(t)$, he showed that, for any fixed $t < \infty$,

$$\left\{\sqrt{n}\,\frac{M_n(s) - M(s)}{\sigma_n(t)} : 0 \leq s \leq t\right\} \Rightarrow_w W\left[\frac{\sigma(s)}{\sigma(t)}\right] \text{ on } [0,\ t],$$

where W is a standard Brownian motion. We use this result to define the test statistic

$$\Theta_n = \sqrt{n}\,\sup_{0 \leq s \leq t} \frac{M_n(0) - M_n(s)}{\sigma_n(t)} \text{ for some } t < X_n.$$

Note that $\sup_{0 \leq s \leq t} [M(0) - M(s)] = 0$ under H_0 while it is positive under the alternative. Thus, we reject H_0 for large values of Θ_n.

2.2 Consistency

We have

$$\begin{aligned}\Theta_n(t) &= \sqrt{n}\sup_{0\le s\le t}\frac{M_n(0)-M_n(s)}{\sigma_n(t)} \text{ for some } t<X_n.\\ &= \sqrt{n}\sup_{0\le s\le t}\frac{1}{\sigma_n(t)}\left[(M_n(0)-M(0))-(M_n(s)-M(s))+(M(0)-M(s))\right]\\ &= \sup_{0\le s\le t}\frac{1}{\sigma_n(t)}\left[B_n(0)-B_n(s)+\sqrt{n}\,(M(0)-M(s))\right]. \qquad (2.1)\end{aligned}$$

If F is under H_0, the last term in (2.1) is zero for all $s \le t$, but if F is in the NBUE class, there exists s_0 in $[0,\ b]$ for some b such that $F(b) < 1$, with $M(0) - M(s_0) > 0$. Hence, $\sup_{0\le s\le t}\frac{1}{\sigma_n(t)}\sqrt{n}\,[M(0)-M(s_0)] \to \infty$, and $\Theta_n(t) \to \infty$, provided that F is NBUE. Thus, our test is consistent.

2.3 Limiting distribution of Θ_n

Theorem 2. *Under H_0,*

$$\Theta_n \to_d \Theta = \sup_{0\le s\le t}\left\{W\left[\frac{\sigma(0)}{\sigma(t)}\right]-W\left[\frac{\sigma(s)}{\sigma(t)}\right]\right\} = W\left[\frac{\sigma(0)}{\sigma(t)}\right]-\inf_{0\le s\le t}W\left[\frac{\sigma(s)}{\sigma(t)}\right]\equiv U-V. \qquad (2.2)$$

Proof. This is obvious from Kumazawa's result above and the continuous mapping theorem.

With the change of variable $u = \frac{\sigma(s)}{\sigma(t)}$, we can write

$$V = \inf_{\sigma(0)/\sigma(t)<u\le 1} W(u).$$

Note that U and V are independent from the independent increments of W and

$$U \sim N(0, \sigma(0)/\sigma(t)).$$

Now, for any $c > 0$,

$$\begin{aligned}P(U-V>c) &= \int P(U-V>c|U=u)d\Phi(u/[\sigma(0)/\sigma(t)])\\ &= \int P(u-V>c)d\Phi(u/[\sigma(0)/\sigma(t)])\\ &= \int P(\inf_{\sigma(0)/\sigma(t)<u\le 1} W(u)\le u-c)d\Phi(u/[\sigma(0)/\sigma(t)])\\ &\le \int P(\inf_{0<u\le 1} W(u)\le u-c)d\Phi(u/[\sigma(0)/\sigma(t)])\\ &= \int_{u>c} 2\bar{\Phi}(c-u)d\Phi(u/[\sigma(0)/\sigma(t)]) \qquad (2.3)\end{aligned}$$

where $\Phi = 1 - \bar{\Phi}$ is the standard normal DF and ϕ is its density; the last equality follows from the fact that

$$P(\inf_{0\leq u\leq 1} W(u) < r) = 2\bar{\Phi}(-r) \text{ for } r \leq 0 \text{ and } 0 \text{ if } r > 0.$$

The integral can be approximated numerically using the estimate of $\sigma(0)/\sigma(t)$ to find the p-value if c is the value of the test statistic. □

3 Testing the exponential distribution versus the DMRL distribution

Let X be a lifetime random variable with continuous distribution function (DF) F where $F(0) = 0$. In this section we consider the problem of testing $H_0 : F(t) = 1 - e^{-\lambda t}$ for $t \geq 0,\ \lambda > 0$ unspecified, versus $H_1 : F$ is in DMRL, not exponential. This is equivalent to $H_0 : M(t) = \frac{1}{\lambda}$ for all $t \geq 0,\ \lambda > 0$ unspecified, but fixed, versus $H_1 : M(s) \geq M(t)$ for all $s \leq t$.

3.1 The test statistic and its properties

We consider the following parameter Θ as a weighted measure of the deviation from H_0 to H_1 :

$$\Theta = \sup_{t\geq 0} \sup_{s\leq t} S(s)S(t)\, [M(s) - M(t)].$$

Clearly, $\Theta = 0$ under H_0. If H_1 is true, M is non increasing and not constant on $[0,\ \infty)$. Thus M has at least one point of decrease, and hence $\Theta > 0$ if H_1 is true. Employing the same sampling plan, our test statistic is the sample analogue of Θ given by

$$\sqrt{n}\, \Theta_n \equiv \sup_{t\geq 0} \sup_{s\leq t} \sqrt{n}\, S_n(s)S_n(t)\, [M_n(s) - M_n(t)].$$

In order to make our test statistic scale invariant we set $\Theta_n^* \equiv \frac{\Theta_n}{M_n(0)}$. Therefore, we can assume that H_0 corresponds to $F,\ Exp(\lambda = 1)$.

3.2 Consistency

We note that

$$\begin{aligned}
\sqrt{n}\, \Theta_n^*(t) &= \sqrt{n} \sup_{t\geq 0} \sup_{s\leq t} S_n(s)S_n(t)[M_n(s) - M_n(t)]/M_n(0) \\
&= \sqrt{n} \sup_{t\geq 0} \sup_{s\leq t}\ S_n(s)S_n(t)[(M_n(s) - M(s)) - (M_n(t) - M(t)) \\
&\qquad +(M(s) - M(t))]/M_n(0).
\end{aligned}$$

If F is distributed as $Exp(1)$, the last term in the previous expression is zero for all $t > 0$. However, if F is in the DMRL class there exists $t_0 \in [0,\ b]$ for some b such that $F(b) < 1$, with $\sup_{s\leq t} S(s)\, S(t_0)[M(s) - M(t_0) > 0$. Hence $\sqrt{n}\, \Theta_n^*(t) \to \infty$ as $n \to \infty,$ provided that F is DMRL.

3.3 Weak convergence

Under H_0, (i) $M(t)) = 1, (ii)$ $h(t) = M(t)$ $S(t) = e^{-t}, (iii)$ $F(s) = 1 - e^{-s}$, and $dF(s) = e^{-s}ds, (iv)$ $C(t) = \int_0^t \frac{e^u}{1-G(u)}\,du$, and$(v)$ $C'(t) = \frac{e^t}{1-G(t)}$. Recall that the MRL process, $\{Z_n(t) = \sqrt{n}\,[M_n(t) - M(t)] : t \in [0,\ b]\} \Rightarrow_w \{B(t) : t \in [0,\ b],\ F(b) < 1\}$, where $\{B(t) : t \in [0,\ b],\ F(b) < 1\}$ is a mean zero Gaussian process. Under H_0,

$$B(t) = Z(t) - \frac{\int_t^{\tau_H} Z(s)e^{-s}\,ds}{e^{-t}},$$

where $\{Z(t) : t \in [0,\ b],\ F(b) < 1\}$ is a Gaussian process with zero mean and covariance function:

$$cov\{Z(s),\ Z(t)\} = C\{\min(s,\ t)\}.$$

Now,

$$cov\{B(s),\ B(t)\} = e^{t+s}\int_t^{\tau_H} \frac{e^{-u}}{1-G(u)}\,du \text{ for } 0 \le s \le t \le \tau_t.$$

Let $0 \le t \le \tau_H$ be fixed and $s \le t$. Note that

$$\begin{aligned}
Z_n(t) &= \sqrt{n}\,[M_n(s) - M_n(t)]\\
&= \sqrt{n}\,[(M_n(s) - M(s)) - (M_n(t) - M(t)) + (M(s) - M(t))]\\
&\Rightarrow_w Z(s) - Z(t) - e^s \int_s^{\tau_H} Z(u)e^{-u}\,du + e^t \int_t^{\tau_H} Z(u)e^{-u}\,du.
\end{aligned}$$

For fixed t, using Slutsky's theorem, $S_n(s)\ S_n(t)\ \sqrt{n}\,[M_n(s) - M_n(t)] \Rightarrow_w Z_t^*(s)$, where

$$Z_t^*(s) = e^{-(s+t)}\,(Z(s) - Z(t)) - e^{-t} \int_s^{\tau_H} Z(u)e^{-u}\,du + e^{-s} \int_t^{\tau_H} Z(u)e^{-u}\,du.$$

By the continuous mapping theorem,

$$\sqrt{n}\,\Theta_n^*(t) \Rightarrow_w \sup_{t\ge 0}\ \sup_{s\le t}\ Z_t^*(s).$$

Although the limiting distribution is intractable we propose a resampling scheme as given in Lin (1997) assuming that $\tau_H = \infty$, which is reasonable since $\tau_F = \infty$ under H_0.

We first note that, from the integration by parts formula, the limiting distribution can be written as

$$\begin{aligned}
Z_t^*(s) &= e^{-t}\left[e^{-s}Z(s) - \int_s^{\infty} Z(u)e^{-u}du\right] - e^{-s}\left[e^{-t}Z(t) - \int_t^{\infty} Z(u)e^{-u}du\right]\\
&= -e^{-t}\int_s^{\infty} e^{-u}dZ(u) + e^{-s}\int_t^{\infty} e^{-u}dZ(u).
\end{aligned}$$

From Kumazawa (1987),

$$Z_n(t) = \sqrt{n}\,\frac{F_n(t) - F(t)}{S_n(t)} = \sqrt{n}\,\int_0^t \frac{S_n(u^-)}{S_n(u)Y_n(u)}dM_n(u) \to_w Z(t) \text{ on } [0, \infty),$$

where

$$Y_n(u) = \sum_{i=1}^{n} Y_{in}(u) = \sum_{i=1}^{n} I(X_i \geq u)$$

and

$$M_n(u) = \sum_{i=1}^{n} M_{in}(u) = N_n(u) - \int_0^u \frac{Y_n(v)}{S_n(v)} dF_n(v) = \sum_{i=1}^{n} \left[N_{in}(u) - \int_0^u \frac{Y_{in}(v)}{S_n(v)} dF_n(v) \right],$$

where $N_{in}(u) = I(X_i \leq u, \delta_i = 1)$. By replacing $M_n(u)$ by $\sum_{i=1}^{n}[G_{in}N_{in}(u)]$, where the G_{in}'s are independent standard normals, the arguments in Lin (1997) show that the distribution of $Z_t^*(\cdot)$ could be approximated for large n by replacing $dZ(u)$ by

$$d\hat{Z}_{n(u)} = \sqrt{n} \sum_{i=1}^{n} \frac{S_n(u^-)}{S_n(u)} G_{in} N_{in}(u)$$

from a large number of realizations from $d\hat{Z}_{n(\cdot)}$ by repeatedly generating $\{G_{in} : 1 \leq i \leq n\}$ while fixing the data $\{S_n(\cdot), Y_{in}, N_{in}(u)\}$. Since we reject H_0 for large values of the test statistic, the p-value could be estimated from the fraction of the generated values under H_0 that exceed the observed value of the test statistic.

3.4 An example

Table 1: Survival times and withdrawal times in months for 211 patients (with number of ties given in parentheses)

Survival times: 0(3), 2, 3, 4, 6, 7(2), 8, 9(2), 11(3), 12(3), 15(2), 16(3), 17(2), 18, 19(2), 20, 21, 22(2), 23, 24, 25(2), 26(3), 27(2), 28(2), 29(2), 30, 31, 32(3), 33(2), 34, 35, 36, 37(2), 38, 40, 41(2), 42(2), 43, 45(3), 46, 47(2), 48(2), 51, 53(2), 54(2), 57, 60, 61, 62(2), 67, 69, 87, 97(2), 100, 145, 158. Withdrawal times: 0(6), 1(5), 2(4), 3(3), 4, 6(5), 7(5), 8, 9(2), 10, 11, 12(3), 13(3), 14(2), 15(2), 16, 17(2), 18(2), 19(3), 21, 23, 25, 27, 28, 31, 32, 34, 35, 37, 38(4), 39(2), 44(3), 46, 47, 48, 49, 50, 53(2), 55, 56, 59, 61, 62, 65, 66(2), 72(2), 74, 78, 79, 81, 89, 93, 99, 102, 104(2), 106, 109, 119(2), 125, 127, 129, 131, 133(2), 135, 136(2), 138, 141, 142, 143, 144, 148, 160, 164(3).

The data in Table 1 were analyzed by Hollander and Proschan (1979) and are an updated version of data considered by Koziol and Green(1976). The data correspond to 211 State IV prostate cancer patients treated with estrogen in a study by the Veterans Administration Cooperative Urological Research Group (1967). By the March 1977 closing date, 90 patients had died of prostate cancer, 105 had died of other diseases, and 16 were still alive. The later 121 observations are treated as censored observations(withdrawals). The same date were also used by Chen, Hollander, and Langberg (1983) to test against monotone MRL.

We calculate the estimates of $\sigma(0)/\sigma(t)$ for some $t < 158$, obtaining the value of our test statistic Θ_n = 1.893. The numerical approximation of the integral in (2.3) for $c = 1.893$ gives the value of 0.035, which is one-sided p value. Thus, with this objective analysis, we find that the test suggests wear-out in the NBUE direction with p value smaller than the p value (= 0.064) obtained by the test statistic of Chen, Hollander, and Langberg (1983).

For the DMRL alternative, the test statistic $\Theta_n^* = 1.772$. Employing resampling scheme suggested by Lin(1997), the estimate of the p value = 0.072, which just replicates the conclusion derived by the test statistic of Chen, Hollander, and Langberg (1983).

4 Conclusion

The test statistic we have proposed for the NBUE class has shown all the properties of a good test statistic. It's consistent and its asymptotic distribution is normal. The test statistic proposed for the DMRL class is also consistent, but its asymptotic distribution is intractable. However, we have proposed a resampling scheme as given in Lin (1997).

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References

[1] Chen, Y. Y., Hollander, M. and Langberg, N. (1983). Tests for monotone mean residual life, using randomly censored data. *Biometrika,* **73***, 119-127.*

[2] Gill, R. (1980). Censoring and stochastic integral. *Mathematical Centre Tracts 124, Mathematisch Centrum, Amsterdam*.

[3] Gross, A. J. and Clark, V. A. (1975). Survival Distributions: Reliability Applications in the Biomedical Sciences. *Wiley, New York.*

[4] Hollander, M. and Proschan, F. (1975). Tests for the mean residual life. *Biometrika,* **62***, 585-593.*

[5] Hollander, M. and Proschan, F. (1979). Testing to determine the underlying distribution using randomly censored data. *Biometrika,* **35***, 393-401.*

[6] Kaplan, E. L. and Meier, P. (1958). Nonparametric estimator from incomplete observations. *J. Amer. Statist.,* **53***, 457-481.*

[7] Koul, H. L. (1978). Testing for new better than used in expectation. *Communications in Statistics,* **7***, 685-701.*

[8] KOUL, H. L. and SUSARLA, V. (1980). Testing for new better than used in expectation with incomplete data. *J. of the American Statistical Association,* **75***, Theory and Methods Section*.

[9] Koziol, J.A. and Green, S. B. (1976). A Cramer-von Mises statistic for tests for randomly censored data. *Biometrika,* **63***, 465-474.*

[10] Kumazawa, Y. (1986a). Tests for new better than used in expectation with randomly censored data. *Sequential Analysis,* **5***, 85-92.*

[11] Kumazawa, Y. (1986b). A class of tests for new better than used in expectation with incomplete data. *Sequential Analysis,* **5(4)***, 347-361.*

[12] Kumazawa, Y. (1987). A note on an estimator of life expectancy with random censorship. *Biometrika,* **74***, 655-658.*

[13] Kuo, W. (1984). Reliability enhancement through optimal burn-in. *IEEE Trans. Reliability,* **33***, 145-156.*

[14] Lim, J. H. and Koh, J. S. (1996). On testing monotonicity of mean residual life from randomly censored data. *ETRI Journal,* **18(3)***, 207-213.*

[15] Lim, J. H and Park, D. H. (1993). Test for DMRL using censored data. *Nonparametric Statistics,* **3***, 167-173.*

[16] Lin, D. Y. (1997). Non-parametric inference for cumulative incidence functions in competing risks studies. *Statistical in Medicine,* **16***, 901-910.*

[17] Veterans Administration Cooperative Urological Research Group (1967). *Treatment and survival of patients with cancer of the prostate. Surgery, Gynecology, and Obstetrics,* **124***, 1011-1017.*

[18] Yang, G. L. (1977). Life expectancy under random censorship. *Stochastic Processes and their Applications,* **6***, 33-39*.